



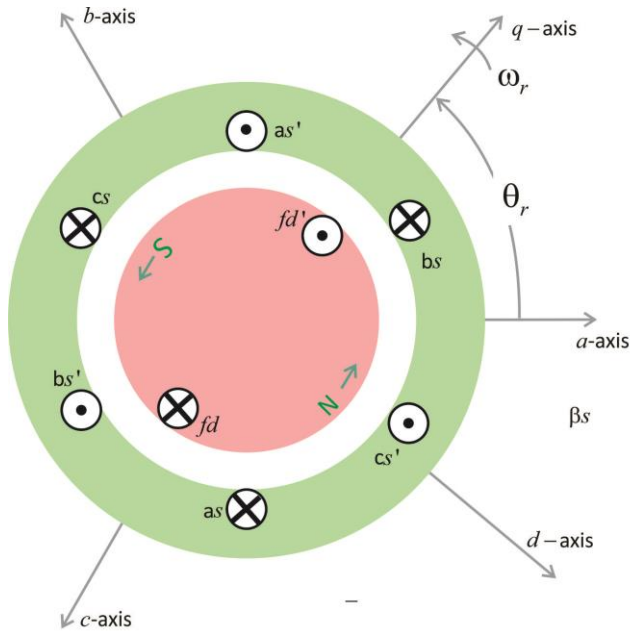
Virtual Synchronous Machine VSG



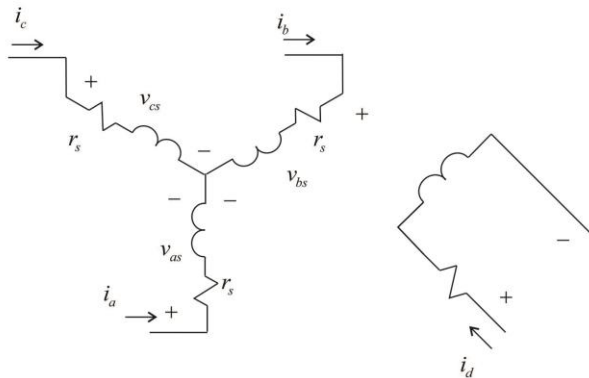
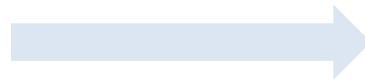
Virtual Synchronous Machine

- ☐ To make the interaction between grid and generator as in a remote power dispatch.
- ☐ To make the reaction to transients as well as the full electrical effects of a rotating mass
- ☐ To provide primary reserve
- ☐ From the grid point of view, to make wind and PV conversion systems to be regarded as conventional power stations
- ☐ To make the inverter based DER to exhibit inertia and damping properties similar of conventional synchronous machines for short time interval

Synchronous Machine



$$\mathbf{K}_s = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$



$$\mathbf{v}_{\alpha\beta} = r_s \mathbf{i}_{\alpha\beta} + \frac{d\lambda_{\alpha\beta}}{dt}$$

$$\lambda_{\alpha\beta} = \begin{bmatrix} \mathbf{L}_s & \mathbf{l}_{sr} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{\alpha\beta} \\ i_d \end{bmatrix}$$

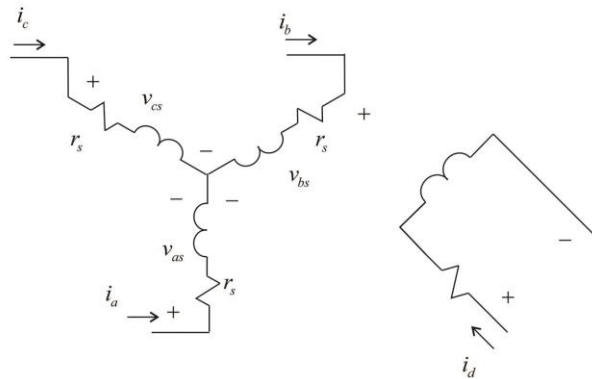
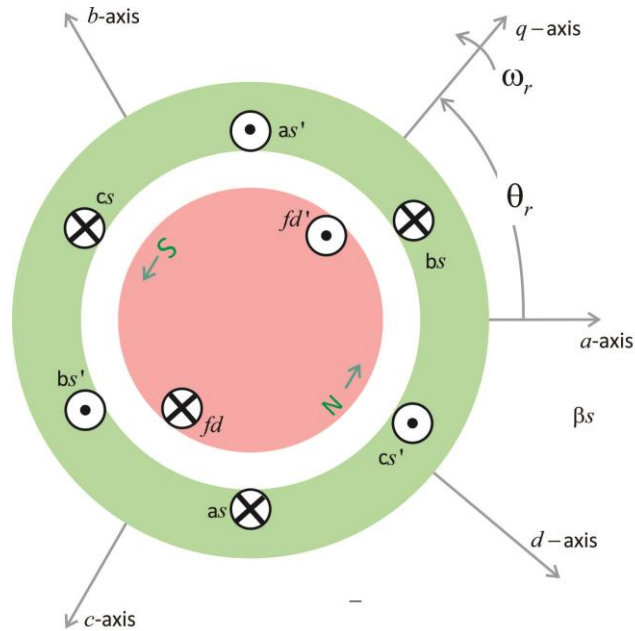
$$\mathbf{L}_s = \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \quad \mathbf{l}_{sr} = \begin{bmatrix} L_{sfd} \sin(\theta_r) \\ -L_{sfd} \cos(\theta_r) \end{bmatrix}$$

$$\mathbf{v}_{\alpha\beta} = r_s \mathbf{i}_{\alpha\beta} + \mathbf{L}_s \frac{d\mathbf{i}_{\alpha\beta}}{dt} + \mathbf{e}_{\alpha\beta}$$

$$\mathbf{e}_{\alpha\beta} = \psi_d \omega_r \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix} \quad \psi_d = L_{sfd} i_d$$

$$T_e = \mathbf{i}_{\alpha\beta}^T \frac{\partial \mathbf{l}_{sr}}{\partial \theta_r} i_d = \psi_d \mathbf{i}_{\alpha\beta}^T \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}$$

Synchronous Machine

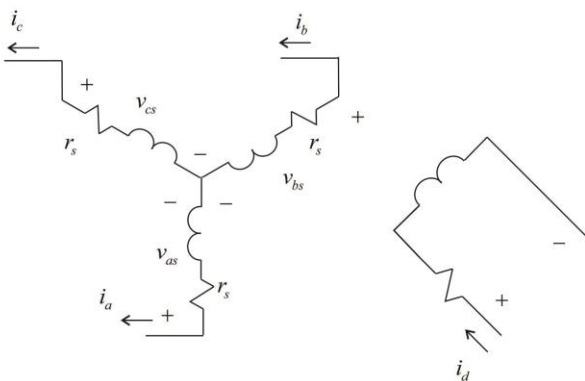
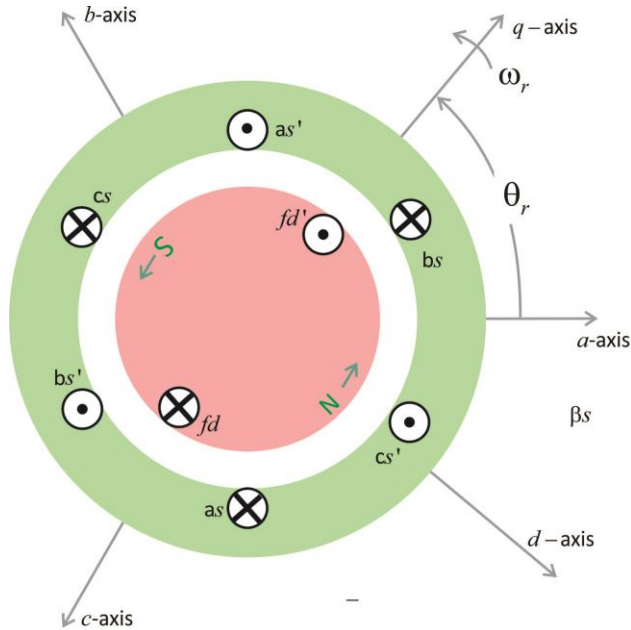


$$\mathbf{v}_{\alpha\beta} = r_s \mathbf{i}_{\alpha\beta} + \mathbf{L}_s \frac{d\mathbf{i}_{\alpha\beta}}{dt} + \mathbf{e}_{\alpha\beta}$$

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$$T_e = \psi_d \mathbf{i}_{\alpha\beta}^T \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}$$

Synchronous Generator



$$\mathbf{v}_{\alpha\beta} = -r_s \mathbf{i}_{\alpha\beta} - \mathbf{L}_s \frac{d\mathbf{i}_{\alpha\beta}}{dt} + \mathbf{e}_{\alpha\beta}$$

$$\mathbf{e}_{\alpha\beta} = \psi_d \omega_r \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}$$

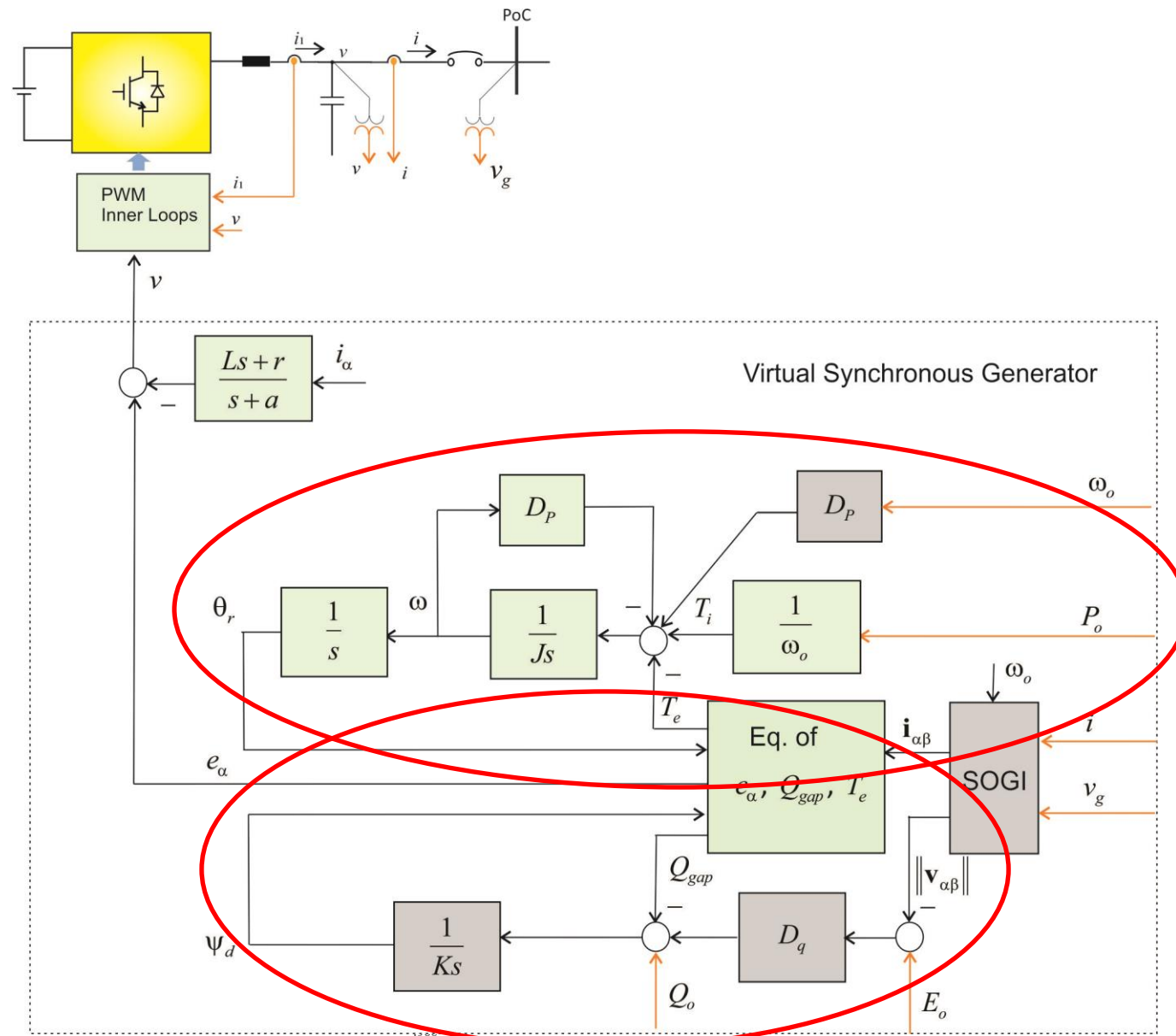
$$T_e = \psi_d \mathbf{i}_{\alpha\beta}^T \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix}$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} (T_I - T_e - D_p \omega_r) \quad \frac{d\theta_r}{dt} = \omega_r$$

$$Q_{gap} = \psi_d \omega_r \mathbf{i}_{\alpha\beta}^T \begin{bmatrix} -\sin(\theta_r) \\ \cos(\theta_r) \end{bmatrix}$$

$$Q_{POC} = \left(\mathbf{v}_{\alpha\beta} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right)^T \mathbf{i}_{\alpha\beta}$$

Virtual Synchronous Generator



Dynamic model – Single phase equivalent VSG connected to a stiff grid

- ❑ The non-linear dynamic equation that describes the behavior of the frequency, power angle, and flux is:

$$\frac{d\omega}{dt} = \frac{1}{J} \left(-D_p \omega - \frac{V\psi \sin(\delta)}{XL} + \frac{P_o}{\omega_o} + D_p \omega_o \right)$$

$$\frac{d\delta}{dt} = \omega - \omega_g$$

$$\frac{d\psi}{dt} = \frac{1}{K} \left(-D_q V + Q_o - \frac{\omega^2 \psi^2}{XL} + \frac{\omega \psi V}{XL} \cos(\delta) + D_q E_o \right)$$

Dynamic model – VSG connected to a Stiff Grid

- The non-linear dynamic equation that describe the behavior of the, frequency, power angle, and flux of the VSG is:

$$\frac{d\omega}{dt} = a_{11}\omega + a_{12}\psi \sin(\delta) + b_{11}$$

$$\frac{d\delta}{dt} = \omega + b_{12}$$

$$\frac{d\psi}{dt} = a_{32}\omega^2\psi^2 + a_{33}\omega\psi \cos(\delta) + b_{31}$$

$$a_{11} = \frac{-D_p}{J}$$

$$a_{12} = \frac{-V}{JX_L}$$

$$a_{32} = \frac{-1}{KX_L}$$

$$a_{33} = \frac{V}{KX_L}$$

$$b_{11} = \frac{1}{J} \left(\frac{P_o}{\omega_o} + D_p \omega_o \right)$$

$$b_{21} = -\omega_g$$

$$b_{31} = \frac{1}{K} (Q_o + D_q (E_o - V))$$

Dynamic model – VSG connected to the Grid

$$0 = a_{11}\omega + a_{12}\psi \sin(\delta) + b_{11}$$

$$0 = \omega + b_{12}$$

$$0 = a_{32}\omega^2\psi^2 + a_{33}\omega\psi \cos(\delta) + b_{31}$$

This algebraic equation has an solutions:

$$(\omega_o, \delta_o, \psi_o)$$

The local qualitative behavior of the VSG can be analyzed from the eigenvalues of:

$$\mathbf{J} = \begin{bmatrix} a_{11} & a_{12}\psi_o \cos(\delta_o) & a_{12} \sin(\delta_o) \\ 1 & 0 & 0 \\ 2a_{32}\omega_o\psi_o^2 + a_{33}\psi_o \cos(\delta_o) & -a_{33}\psi_o\omega_o \sin(\delta_o) & 2a_{32}\omega_o^2\psi_o + a_{33}\omega_o \cos(\delta_o) \end{bmatrix}$$

Example

Let us consider again a 33kVA single phase inverter connected to a 220V 60Hz grid where:

First, the droop coefficients can be found as:

$$D_p = \frac{\Delta T}{\Delta \omega} = \frac{\Delta P}{\omega_g \Delta \omega} = \frac{33000}{(2\pi 60)(2\pi 3)} = 4.64$$

$$D_q = \frac{\Delta Q}{\Delta E} = \frac{33000}{0.1(220)} = 1500$$

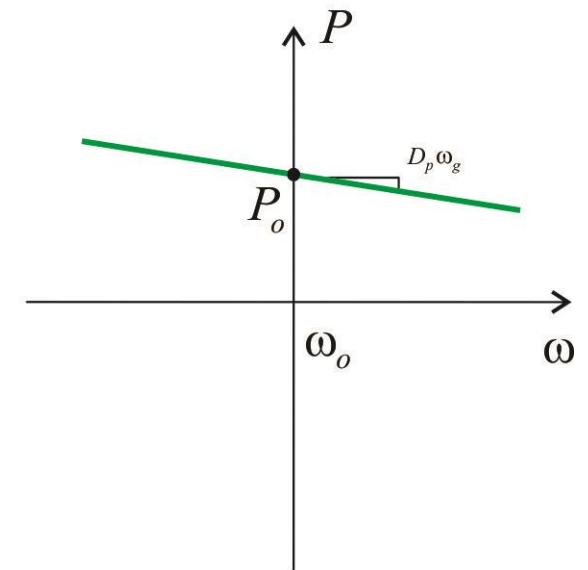
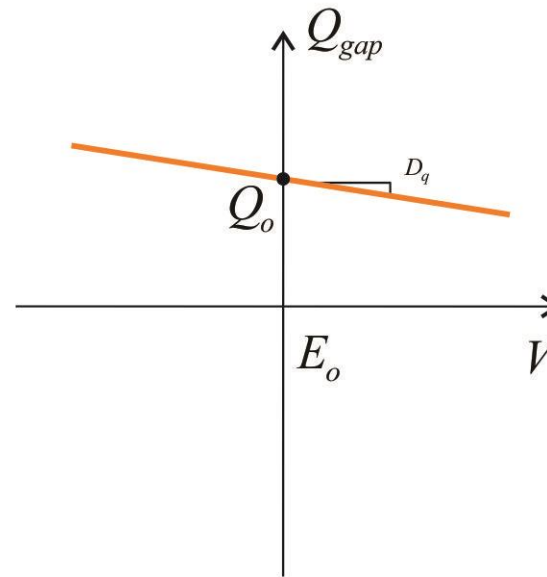
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Example

The time constants associated with the frequency and the voltage are:

$$\tau_f = \frac{J}{D_p}$$



$$J = \tau_f D_p$$

$$K = \tau_v \omega_g D_q$$

$$\tau_v = \frac{K}{\omega_g D_q}$$

By selecting

$$\tau_f = 0.1$$

$$\tau_v = 0.5$$



$$J = 0.464$$

$$K = 282743.34$$

Qualitative Behavior

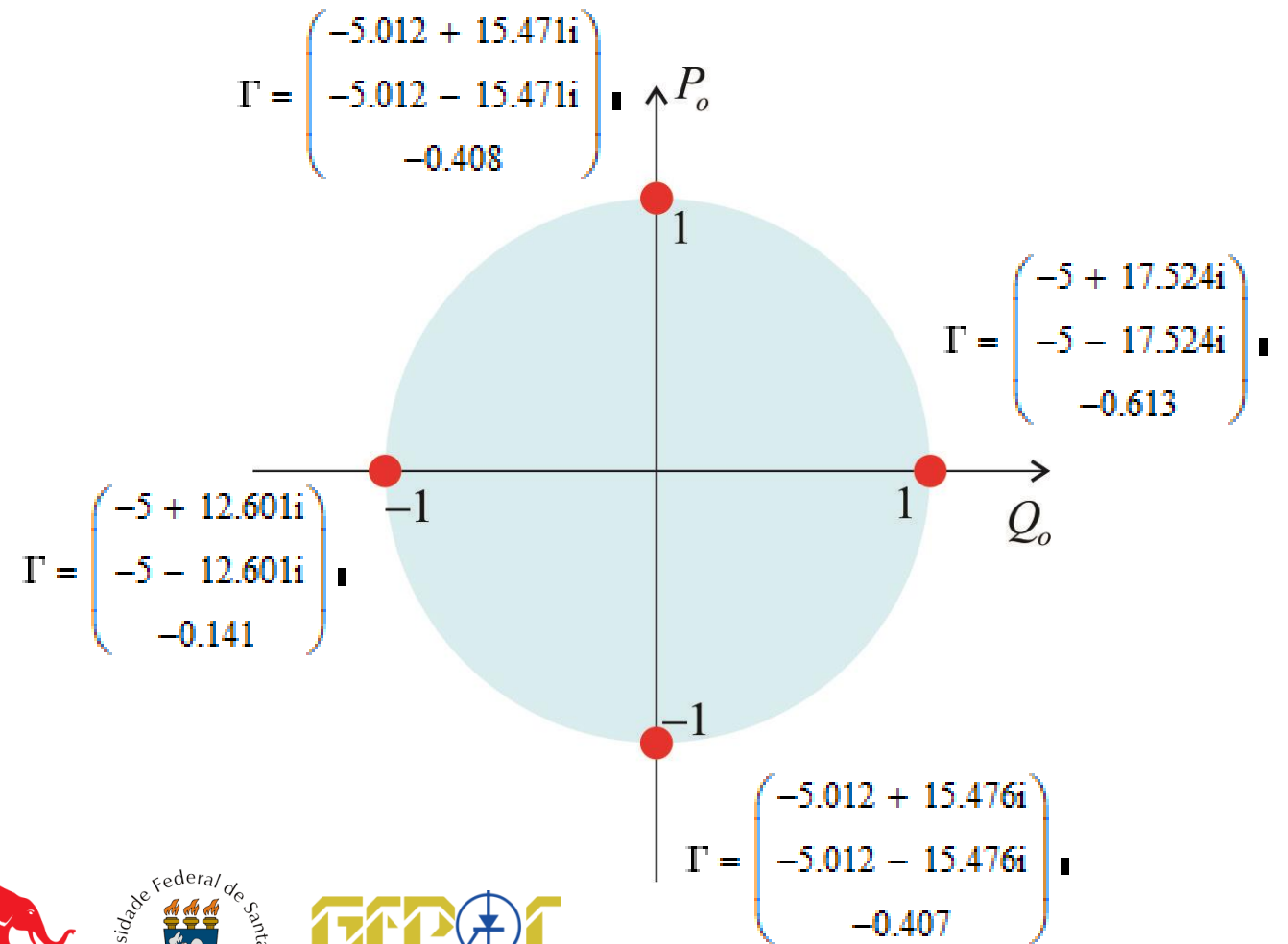
$$D_p = 4.64$$

$$D_q = 1500$$

$$J = 0.464$$

$$K = 113110$$

$$X_L = 0.257(\text{pu})$$



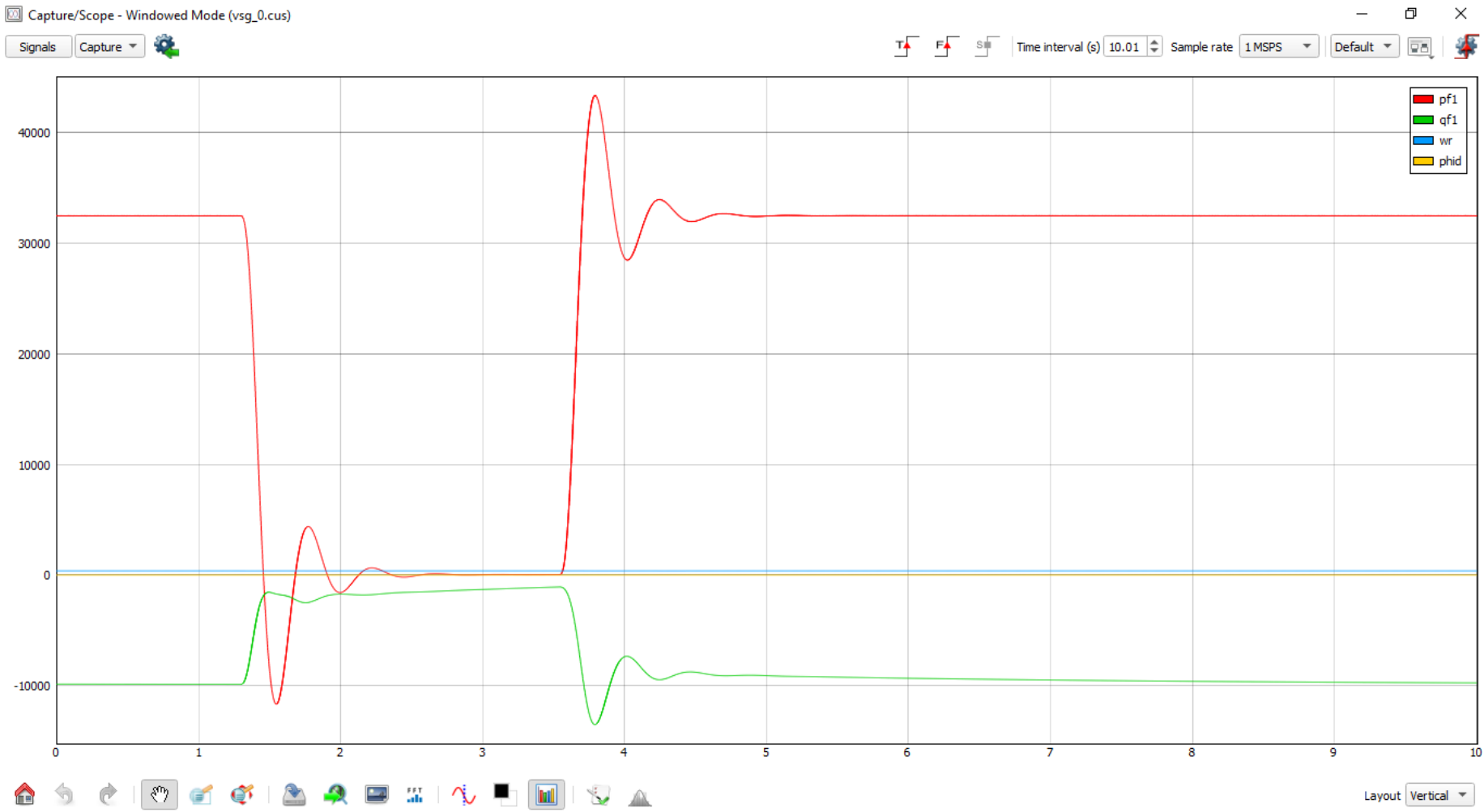
Example

The relation between the states and the modes can be revealed from the participation factors.

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} -5.01 + j15.47 \\ -5.01 - j15.47 \\ -0.41 \end{bmatrix}$$

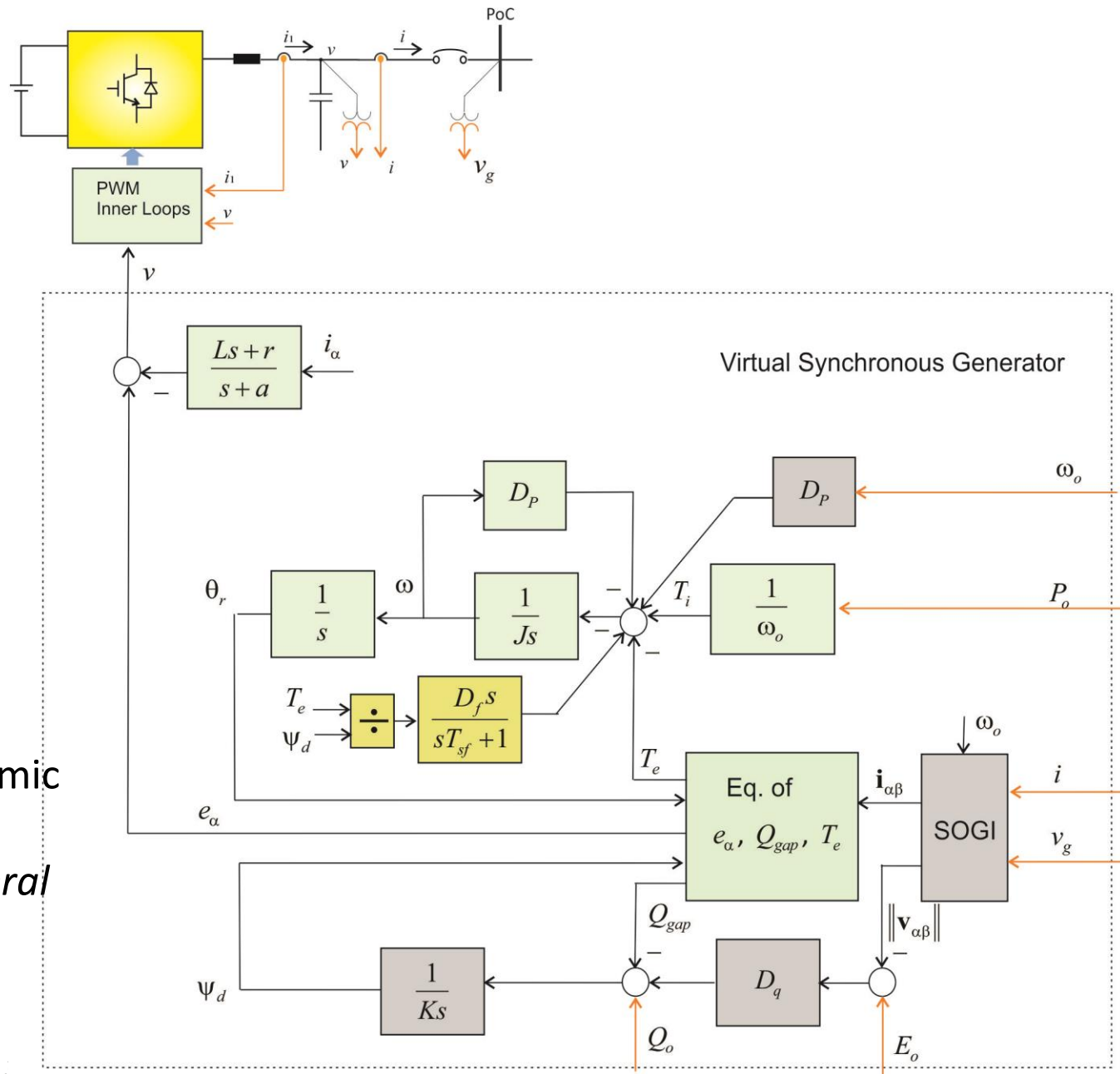
λ_1	λ_2	λ_3	
0.525	0.525	0	ω
0.525	0.525	0	δ
0	0	0.999	ψ

Simulation



$$D_f \frac{d}{dt} \left(\frac{T_e}{\psi_d} \right) \propto (\omega - \omega_g)$$

S. Dong and C. Chen, "Adjusting Synchronverter Dynamic Response Speed via Damping Correction Loop," published in *2018 IEEE Power & Energy Society General Meeting (PESGM)*, Portland, OR, 2018, pp. 1-1"



Damping Correction Action



Simulation

